I PUC – MATHEMATICS
CHAPTER - 08

Binomial Theorem

Two Marks Questions

1. Expand $\left(x+\frac{1}{2y}\right)^5$ using binomial theorem and hence find the coefficient of $\frac{x^2}{y^3}$
2. Expand $\left(\frac{2}{x}-x\right)^4$ using binomial theorem. Hence find the constant term of the expansion.
3. Simplify $(\sqrt{3}+1)^6+(\sqrt{3}-1)^6$
4. Simplify $(\sqrt{5}+1)^4+(\sqrt{5}-1)^4$
5. Using binomial theorem, evaluate $(97)^5$
6. Using binomial theorem evaluate $(0.99)^6$ correct to four decimal places.
7. Using binomial theorem evaluate $(10.2)^4$
8. Find the 7th term in the expansion of $\left(\frac{2x^2}{3}-\frac{3}{2x}\right)^{11}$
9. Find the 12th term in the expansion of $\left(\frac{2}{y}-x\right)^{20}$
10. Find the coefficient of x^2 in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x}\right)^{10}$
11. Find the coefficient of $\frac{b^3}{a^5}$ in the expansion of $\left(\frac{2}{a}+3b\right)^8$
12. Find the ratio of the coefficient of x^8 to the coefficient of x^4 in the expansion of $(1-x^2)^{10}$.
13. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^6$ is $\frac{5}{2}$, evaluate 'a'
14. Find the middle term in the expansion of $\left(x^2+\frac{3}{x}\right)^{20}$.
15. Find the middle term in the expansion of $\left(\frac{y}{b}-\frac{b}{y}\right)^{12}$
16. Find the middle term in the expansion of $\left(2a-\frac{a^3}{6}\right)^{10}$
17. Find the term independent of 'x' in the expansion of $\left(\frac{x^2}{2}-\frac{1}{3x^3}\right)^{10}$
18. Find the constant term in the expansion of $\left(\frac{2}{x^2}-\sqrt{x}\right)^{15}$

19. Show that there is no term independent of x in the expansion of $\left(5x\sqrt{x}-\frac{2}{\sqrt{x}}\right)^{30}$

20. Prove that there is no term involving x^5 in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$

21. Prove that the coefficients of x^m and x^n are equal in the expansion of $(1+x)^{m+n}$ where 'm' and 'n' are positive integers.

5 mark questions:

1. The 3rd , 4th and 5th terms in the expansion of $(x+a)^n$ are respectively 84, 280 and 560. Find the values of x, a and n.
2. Using binomial theorem prove that $3^{2n} - 8n - 9$ is divisible by 8 where 'n' is a positive integer.
3. The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42 find 'n'.
4. Find the middle terms in the expansion of $\left(\frac{5}{2x}+\frac{4x}{5}\right)^9$.
5. Find the coefficient of x^7 in $(1 + x + x^4 + x^5)^6$. .
6. Find the coefficient of x^{10} in the expansion of $(1+x+x^2)(1 - x)^{15}$.
7. Show that the coefficients of x^5 in the expansion of $(1+x^2)^5 (1+x)^4$ is 60.
8. Given that the coefficients of (2m +1)th and (m+2)th terms in the expansion of $(1+x)^{43}$ are equal, find 'm'
9. Find the term independent of x in the expansion of $(1 - 2x + x^3)\left(x-\frac{1}{x}\right)^{15}$

Solutions

Binomial Theorem

Solution for two marks questions.

1. $\left(x+\frac{1}{2y}\right)^5 = x^5 + {}^5C_1\, x^4 \left(\frac{1}{2y}\right) + {}^5C_2\, x^3 \left(\frac{1}{2y}\right)^2 + {}^5C_3\, x^2\left(\frac{1}{2y}\right)^3 + {}^5C_4\, x \left(\frac{1}{2y}\right)^4 + \left(\frac{1}{2y}\right)^5$

$= x^5 + \frac{5x^4}{2y} + \frac{10x^3}{4y^2} + \frac{10x^2}{8y^3} + \frac{5x}{16y^4} + \frac{1}{32y^5}$

$= x^5 + \frac{5x^4}{2y} + \frac{5x^3}{2y^2} + \frac{5x^2}{4y^3} + \frac{5x}{16y^4} + \frac{1}{32y^5}$

$\therefore$ The coefficient of $\frac{x^2}{y^3}$ is $\frac{5}{4}$.

2. $\left(\frac{2}{x}-x\right)^4 = \left(\frac{2}{x}\right)^4 + {}^4C_1\left(\frac{2}{x}\right)^3(-x) + {}^4C_2\left(\frac{2}{x}\right)^2(-x)^2 + {}^4C_3\left(\frac{2}{x}\right)(-x)^3 + (-x)^4$

$= \frac{16}{x^4} - 4\left(\frac{8}{x^3}\right) + \frac{16}{x^2}(x^2) - \frac{8}{x}(x)^3 + x^4$

$= \frac{16}{x^4} - \frac{32}{x^2} + 24 - 8x^2 + x^4$

The constant term of the expansion = T_3 = 16.

3. Consider

$(\sqrt{3}+1)^6 = \left(\sqrt{3}\right)^6 + {}^6C_1(\sqrt{3})^5 + {}^6C_2(\sqrt{3})^4 + {}^6C_3(\sqrt{3})^3 + {}^6C_4(\sqrt{3})^2 + {}^6C_5(\sqrt{3}) + 1$

$(\sqrt{3}+1)^6 = 33 + 6(9\sqrt{3}) + 15\,(9) + 20\,(3\sqrt{3}) + 15(3) + 6\sqrt{3} + 1$

$(\sqrt{3}-1)^6 = 33 - 54\sqrt{3} + 135 - 60\sqrt{3} + 45 - 6\sqrt{3} + 1$

$(\sqrt{3}+1)^6 + (\sqrt{3}-1)^6 = 2\left(3^3\right) + 2(135) + 2(45) + 2$

$= 2\,[27 + 135 + 45 + 1]$

$= 2[208]$

$= 416$

4. Consider

$(\sqrt{5}+1)^4 = \left(\sqrt{5}\right)^4 + {}^4C_1(\sqrt{5})^3 + {}^4C_2(\sqrt{5})^2 + {}^4C_3(\sqrt{5}) + 1$

$= 25 + 4(5\sqrt{5}) + 6\,(5) + 4\sqrt{5} + 1$

$(\sqrt{5}+1)^4 = 25 + 20\sqrt{5} + 30 + 4\sqrt{5} + 1$

$(\sqrt{5}-1)^4 = 25 - 20\sqrt{5} + 30 - 4\sqrt{5} + 1$

$(\sqrt{5}+1)^4 - (\sqrt{5}-1)^4 = 2\,[20\sqrt{5} + 4\sqrt{5}\,]$

$= 2\{24\sqrt{5}\,]$

$= 48\sqrt{5}$

5. $(97)^5 = (100-3)^5 =$

$= (100)^5 - 5C_1\,(100)^4(3) + 5C_2(100)^3(3)^2 - 5C_3(100)^2\,(3)^3 + 5C_4(100)(3)^4 - (3)^5$

$= 10000000000 - 5(100\ 00\ 00\ 00)\,(3) + 10(100\ 00\ 00)(9)$

$-10(10000)\,(27) + 5(100)\,(81) - 243$

$= 8587340257$

6. $(0.99)6 = (1-0.01)6$

$= 1 - 6C_1(0.01) + 6C_2(0.01)^2 - 6C_3(0.01)^3 + 6C_4(0.01)^4 - 6C_5(0.01)^5 + (0.01)^6$

$= 1 - 0.06 + 15(0.0001) - 20(0.000001) + \ldots\ldots$(neglecting higher powers of 0.01)

$= 0.9415$

1. $(10.2)^4 = (10+0.2)^4$

$= 10^4 + 4C_1(10)^3\,(0.2) + 4C_2(10)^2\,(0.2)^2 + 4C_3(10)(0.2)^3 + + (0.2)^4$

$= 10000 + 4(1000)(0.2) + 6(100)\,(0.04) + 4(10)(0.008) + (0.0016)$

$= 10824.3216$

2. The general term in the expansion of $\left(\frac{2}{3}x^2 - \frac{3}{2x}\right)^{11}$ is

$$T_{r+1} = {}^{11}C_r\left(\frac{2}{3}x^2\right)^{11-r}\left(\frac{-3}{2x}\right)^r$$

Put r = 6, $T_{6+1} = {}^{11}C_6\left(\frac{2}{3}x^2\right)^5\left(\frac{-3}{2x}\right)^6$

$$T_7 = {}^{11}C_6\,\frac{2^5\cdot x^{10}\cdot 3^6}{35\cdot 26\cdot 6}$$

$$= {}^{11}C_6\,.\left(\frac{3}{2}x^4\right) \qquad \left[\therefore\ {}^{11}C_6 = {}^{11}C_5 = \frac{11\times10\times9\times8\times7}{5\times4\times3\times2\times1} = 462\right]$$

$\therefore\ T_7 = 693x^4$

3. $T_{r+1} = {}^{20}C_r\left(\frac{2}{y}\right)^{20-r}(-x)^r$

Put r = 11,

$$T_{11+1} = {}^{20}C_{11}\left(\frac{2}{y}\right)^9(-x)^{11}$$

$$T_{12} = {}^{-20}C_9\,.\ \frac{2^9\cdot x^{11}}{y^9}$$

4. $T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x}\right)^r$

$$= {}^{10}C_r\,\frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}}\cdot\frac{3^r}{2^r\,x^r}$$

$$T_{r+1} = {}^{10}C_r\cdot\frac{3^r\cdot x^{\frac{10-r}{2}-r}}{3^{\frac{10-r}{2}}\cdot 2^r} \qquad \rightarrow (1)$$

If $\frac{10-r}{2} - r = 2$

➔ $10 - 3r = 4$

➔ $r = 2$

Substituting r = 2 in (1) $\quad [{}^{10}C_2 = \frac{10\times9}{2\times1} = 45]$

$T_3 = {}^{10}C_2 \, \frac{3^2 \cdot x^2}{3^4 \cdot 2^2}$

$= \frac{45x^2}{9\times4} = \frac{5x^2}{4}$

$\therefore$ coefficient of x^2 is $\frac{5}{4}$

5. General term = $T_{r+1} = {}^8C_r \left(\frac{2}{a}\right)^{8-r} (3b)^r$

$T_{r+1} = {}^8C_r \, \frac{2^{8-r}}{a^{8-r}} \cdot 3^r (b)^r$

By data, the power of b must be 3. $\quad [{}^8C_3 = \frac{8\times7\times6}{3\times2\times1} = 56]$

$\therefore$ for r = 3

$T_{3+1} = {}^8C_3 \, \frac{2^5}{a^5} \cdot 3^3 (b)^3$

$= (56)\,(32)\,(27)\,\frac{b^3}{a^5}$

$T_4 = 48384 \, \frac{b^3}{a^5}$

$\therefore$ coefficient of $\frac{b^3}{a^5}$ is 48384

6. $T_{r+1} = {}^{10}C_r (1)^{10-r} (-x^2)^r$

$= {}^{10}C_r (-x^2)^r$

Clearly coefficient of x^8 is ${}^{10}C_4$ and the coefficient of x^4 is ${}^{10}C_2$.

$\therefore$ the required ratio $\frac{10C_4}{10C_2} = \frac{\left(\frac{10\times9\times8\times7}{4\times3\times2\times1}\right)}{\left(\frac{10\times9}{2\times1}\right)}$

$= \frac{8\times7}{4\times3} = \frac{14}{3}$

$\therefore$ the ratio is 14 : 3

7. $T_{r+1} = {}^6C_r (ax)^{6-r} \left(\frac{1}{x}\right)^r$

Putting r = 3

$T_{3+1} = {}^6C_3\, a^3\, x^3 \left(\frac{1}{x^3}\right)$ $\quad [{}^6C_3\ \frac{6\times5\times4}{3\times2\times1} = 20]$

$\therefore\ T_4 = 20\, a^3$

Given $T_4 = \frac{5}{2}$

$\therefore\ 20\, a^3 = \frac{5}{2} \Rightarrow a^3 = \frac{1}{8}$

$\Rightarrow a = \frac{1}{2}$

8. $T_{r+1} = {}^{20}C_r\, \left(x^2\right)^{20-r} \left(\frac{3}{x}\right)^r$

Since n = 20 there are 21 terms in the expansion.
T_{11} is the middle term.
Putting r = 10, we get

$T_{11} = {}^{20}C_{10}\, \left(x^2\right)^{20-10} \left(\frac{3}{x}\right)^{10}$

$= {}^{20}C_{10}\, x^{20} \cdot \frac{3^{10}}{x^{10}}$

$T_{11} = {}^{20}C_{10} \cdot 3^{10} \cdot x^{10}$ is the middle term.

9. $T_{r+1} = {}^{12}C_r \left(\frac{y}{b}\right)^{12-r} \left(-\frac{b}{y}\right)^r$

n = 2 $\quad\therefore$ There are 13 terms in the expansion
T_7 is the middle term.
Putting r = 6 we get

$T_{6+1} = {}^{12}C_6 \left(\frac{y}{b}\right)^6 \left(-\frac{b}{y}\right)^6$

$T_7 = {}^{12}C_6\ \frac{y^6}{b^6} \times \frac{b^6}{y^6}$

$T_7 = {}^{12}C_6 = 924$ is the middle term

10. $T_{r+1} = {}^{10}C_r\, (2a)^{10-r} \left(\frac{-a^3}{6}\right)^r$

n = 10 $\quad\therefore$ the middle term is $T_{\frac{n}{2}+1} = T6$

putting r = 5, $T_{5+1} = {}^{10}C_5\, (2a)^{10-5} \left(\frac{-a^3}{6}\right)^5$

$T_6 = {}^{10}C_5\, 2^5 a^5 \left(-\frac{a^{15}}{6^5}\right)$

$= -\ \frac{(252)(2^5)a^{20}}{2^5 \times 3^5}$

$= - \frac{252}{243} a^{20}$ is the middle term

11. $T_{r+1} = {}^{10}C_r \left(\frac{x^2}{2}\right)^{10-r} \left(-\frac{1}{3x^3}\right)^r$

$= {}^{10}C_r \; \frac{x^{20-2r}}{2^{10-r}} \; \frac{(-1)^r}{3^r x^{3r}}$

$T_{r+1} = {}^{10}C_r \; \frac{(-1)^r}{2^{10-r} 3^r} x^{20-5r}$

Equating the power of x to zero.

$20 - 5r = 0$

➔ $r = 4$

From equation (1)

$T_{4+1} = {}^{10}C_4 \; \frac{(-1)4}{2^6 3^4}$ $\qquad {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

$T_5 = \frac{35}{864}$ is the constant term

12. $T_{r+1} = {}^{15}C_r \left(\frac{2}{x^2}\right)^{15-r} (-\sqrt{x})^r$

$= {}^{15}C_r \; \frac{2^{15-r}}{x^{30-2r}} (-1)^r x^{\frac{r}{2}}$

$T_{r+1} = {}^{15}C_r(-1)^r \, 2^{15-r} \quad x^{\frac{r}{2}-30+2r}$

For constant term, $\frac{r}{2} - 30 + 2r = 0$

Solving we get r = 12 $\qquad \therefore {}^{15}C_{12} = {}^{15}C_3$

$= \frac{15\times14\times13}{3\times2\times1} = 455$

$\therefore T_{12+1} = {}^{15}C_{12}(-1)^{12} \, 2^3$

$= 455 \times 8$

$T_{13} = 3640$ is the constant term .

13. $T_{r+1} = {}^{30}C_r \, (5x\sqrt{x})^{30-r} \left(-\frac{2}{\sqrt{x}}\right)^r$

$= {}^{30}C_r \, 5^{30-r} \left(x^{3/2}\right)^{30-r} \frac{(-2)^r}{x^{\frac{r}{2}}}$

$= {}^{30}C_r \, 5^{30-r} \, (-2)^r \; x^{\left(\frac{3(30-r)}{2} - \frac{r}{2}\right)}$

For the constant term, $\frac{90-3r}{2} - \frac{r}{2} = 0$

➔ $90 - 3r - r = 0$

➔ $4r = 90$

➔ $r = \frac{45}{2}$ which is a fraction.

Since the value of r cannot be a fraction, there doesn't exist constant term in the expansion.

14. $T_{r+1} = {}^{15}C_r\,(x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r\, x^{60-4r}\,\frac{(-1)^r}{x^{3r}}$

$T_{r+1} = {}^{15}C_r\,(-1)^r\,x^{60-7r}$

Equating the power of x to 5,

$60 - 7r = 5$

$7r = 55$

$R = \frac{55}{7}$ which is a fraction.

$\therefore$ there doesn't exist term containing x^5 in the expansion.

15. $T_{r+1} = {}^{(m+n)}C_r\,(1)^{m+n-r}\,x^r$

$T_{r+1} = {}^{(m+n)}C_r\,x^r$

Coefficient of x^m is ${}^{(m+n)}C_m$

Coefficient of x^n is ${}^{m+n}\,C_n$

But ${}^{(m+n)}C_m = {}^{(m+n)}C_{m+n-m}$

${}^{(m+n)}C_m = {}^{(m+n)}C_n$

Hence proved.

Solutions for 5 marks questions

1. $T_{r+1} = {}^nC_r\;X^{n-r}\;a^r$

Given $T_3 = 84$, $T_4 = 280$, $T_5 = 560$

Now $T_3 = 84$

- ${}^nC_2\;x^{n-2}\;a^2 = 84$
- $\frac{n(n-1)(x)^{n-2}a^2}{2x1} = 84$
- $n(n-1)\,x^{n-2}\,a^2 = 84 \text{ x } 2 \Rightarrow (1)$

$T_4 = 280$

- ${}^nC_3\;x^{n-3}\;a^3 = 280$
- $\frac{n(n-1)(n-2)}{3\times2\times1}x^{n-3}\,a^3 = 280$
- $n(n-1)\,(n-2)\,x^{n-3}\,a^3 = 280 \text{ x } 6 \Rightarrow (2)$

$T_5 = 560$

- ${}^nC_4\;x^{n-4}\;a^4 = 560$
- $\frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}x^{n-4}\,a^4 = 560$
- $n(n-1)\,(n-2)\,(n-3)\,x^{n-4}\,a^4 = 560 \text{ x } 24 \Rightarrow (3)$

$\frac{(3)}{(2)}$

- $\frac{(n-3)a}{x} = \frac{560\times24}{280\times6}$
- $(n-3)a = 8x \Rightarrow \quad (4)$

$\frac{(2)}{(1)}$

➢ $\frac{(n-2)a}{x} = \frac{280\times 6}{84\times 2}$

➢ (n – 2) a = 10x ➔ (5)

$\frac{(4)}{(5)}$

$\frac{n-3}{n-2} = \frac{4}{5}$

➢ 5(n-3) = 4(n – 2)

➢ n = 7

Putting n = 7 in (4) we get a = 2x

Substituting n = 7 and a = 2x in (1) , we get

$7 \times 6 \times x^{7-2} (2x)^2 = 84 \times 2$

$x^7 = \frac{84\times 2}{7\times 6\times 4}$

➢ $X^7 = 1$

∴ x = 1

Since a = 2x ➔ a = 2

∴ x = 1, a = 2, n = 7

2. Consider $(1 + 8)^n = 1 + {}^nC_1\, 8 + {}^nC_2\, (8)^2 + {}^nC_3\, 8^3 + \ldots\ldots + 8^n$

➢ $9^n = 1 + 8n + {}^nC_2\, (8)^2 + {}^nC_3\, 8^3 + \ldots\ldots + 8^n$

➢ $9^n - 8n - 9 = -8 + {}^nC_2\, (8)^2 + {}^nC_3\, 8^3 + \ldots\ldots + 8^n$

$9^n - 8n - 9 = 8\, [-1 + {}^nC_2\, (8) + {}^nC_3\, 8^2 + \ldots\ldots + 8^{n-2}]$

$9^n - 8n - 9 = 8\,K$

Where $K = -1 + 8(nC_2) + 8^2\, .nC_3 + \ldots\ldots + 8^{n-2}$ is an integer.

∴ $9^n - 8n - 9$ is divisible by 8. i.e. $3^{2n} - 8n - 9$ is divisible by 8

3. $T_{r+1} = {}^nC_r\, x^r$

Let coefficients of T_{r-1}, T_r, T_{r+1} be in the ratio 1 : 7 : 42

$\frac{coeff.ofT_{r-1}}{coeff.ofT_r} = \frac{1}{7}$ and $\frac{coeff.ofT_r}{coeff.ofT_{r+1}} = \frac{7}{42}$

➢ $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{1}{7}$ and $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{1}{6}$

➢ $\frac{r}{n-r+1} = \frac{1}{7}$ and $\frac{r+1}{n-r} = \frac{1}{6}$

➢ 7r = n – r + 1 and $\left[\therefore \frac{{}^nC_{k-1}}{{}^nC_k} = \frac{k}{n-k+1}\right]$

6(r+1) = n – r

➢ 8r = n + 1 and 7r = n – 6

Solving these equations we get

r = 7, n = 55

4. n = 9 ∴There are 10 terms in the expansion.

∴ T_5 and T_6 are the middle terms .

$$T_{r+1} = {}^9C_r \left(\frac{5}{2x}\right)^{9-r}\left(\frac{4x}{5}\right)^r \Rightarrow \quad (1)$$

Putting r = 4 we get,

$$T_{4+1} = {}^9C_4 \left(\frac{5}{2x}\right)^{9-4}\left(\frac{4x}{5}\right)^4$$

$$T_5 = {}^9C_4 \cdot \frac{5^5}{2^5 x^5} \times \frac{4^4 x^4}{5^4}$$

$$T_5 = 126 \times \frac{5(2)^8}{2^5 x}$$

$$= \frac{5040}{x}$$

Putting x = 5 in (1) we get,

$$T_{5+1} = {}^9C_5 \left(\frac{5}{2x}\right)^{9-5}\left(\frac{4x}{5}\right)^5$$

$$T_6 = {}^9C_4 \cdot \frac{5^4}{2^4 x^4} \times \frac{4^5 x^5}{5^5}$$

$$T_6 = 126 \times \frac{(2)^{10} x}{(2^4)(5)}$$

$$T_6 = \frac{8064x}{5}$$

$\therefore \frac{5040}{x}$ and $\frac{8064x}{5}$ are the middle terms.

5. $[1 + x + x^4 + x^5]^6$

$= [1 + x + x^4 (1+x)]^6$

$= (1 + x)^6 (1 + x^4)^6$

$= [1 + 6C_1 x + 6C_2 x^2 + \ldots\ldots\ldots + x^6] [1 + 6C_1 x^4 + 6C_2 x^4)^2 \ldots\ldots\ldots + x^4)^6]$

$\therefore$ coefficient of x^7

$= 6C_3 \times 6C_1$

$= 120$

6. $(1 + x + x^2) (1 - x)^{15} = (1 + x + x^2) (1 - x) (1 - x)^{14}$

$= (1 - x^3) (1 - x)^{14}$

$= (1 - x^3) (1 - {}^{14}C_1 x + {}^{14}C_2 x^2 \ldots\ldots\ldots\ldots\ldots\ldots + x^{14})$

$\therefore$ The coefficient of $x^{10} = {}^{14}C_{10} - {}^{14}C_7$

$= {}^{14}C_4 - {}^{14}C_7$

$= 1001 - 3432$

$= - 2431$

7. $(1 + x^2)^5 (1 + x)^4$

$= [1 + 5C_1 x^2 + 5C_2 (x^2)^2 + \ldots\ldots\ldots\ldots + (x^2)^5] [1 + 4C_1 x + 4C_2 x^2 + \ldots\ldots + x^4]$

Coefficient of $x^5 = 5C_2 \times 4C_1 + 5C_1 \times 4C_3$

$= 10 \times 4 + 5 \times 4$

$= 40 + 20$

$= 60$

8. $T_{r+1} = {}^{43}C_r x^r$

Putting r = 2m,

$T_{2m+1} = {}^{43}C_{2m}\, x^{2m}$

For r = m + 1

$T_{m+2} = {}^{43}C_{m+1}\; x^{m+1}$

Given coefficient of T_{2m+1} = coefficient of T_{m+2}

- ${}^{43}C_{2m} = {}^{43}C_{m+1}$
- 2m = m + 1 or 2m + m + 1 = 43
- m = 1 or m = 14

9. Constant term

$= [-2 \times \text{coefficient of } x^{-1} \text{ in } \left(x - \frac{1}{x}\right)^{15} + [1 \times \text{coefficient of } x^{-3} \text{ in } \left(x - \frac{1}{x}\right)^{15} \Rightarrow (1)$

General term of $\left(x - \frac{1}{x}\right)^{15}$ is $T_{r+1} = {}^{15}C_r\, x^{15-r} \left(\frac{-1}{x}\right)^r$

$T_{r+1} = {}^{15}C_r (-1)^r\, x^{15-2r}$

If 15 – 2r = -1 ➔ r = 8

If 15 – 2r = -3 ➔ r = 9

$\therefore$ coefficient of x^{-1} is ${}^{15}C_8$

And coefficient of x^{-3} is $-{}^{15}C_9$

$\therefore$ from (1)

Constant term = $[-2 \times 15C_8] + [1 \times (-15\, C_9)]$

$= (-2 \times 15C_7\, x) + (-15C_6)$

$= -12870 - 5005 = -17875$

****