MATHEMATICS (STANDARD)

(SOLUTIONS)

SECTION – A

1. **Answer:** (1) **[1]**
2. **Answer:** (2) **[1]**
3. **Answer:** (3) **[1]**
4. **Answer:** (3) **[1]**
5. **Answer:** (2) **[1]**
6. **Answer:** (4) **[1]**
7. **Answer:** (4) **[1]**
8. **Answer:** (4) **[1]**
9. **Answer:** (1) **[1]**
10. **Answer:** (2) **[1]**

11. cos30° cos45° cos60° cos75° cos90° cos105° cos120° cos135°

    = cos30° × cos45° × cos60° × cos75° × 0 × cos105° × cos120° × cos135°

    = 0 **[1]**

12. Let M (x, y) be the midpoint of AB. Then,

    $x = \frac{\{(-5)+7\}}{2} = 1$ and $y = \frac{\{4+(-8)\}}{2} = -2$

    Hence, the required point is M (1, -2). **[1]**

    OR

    Required Coordinate = $\left(\frac{4-4}{2}, \frac{5+7}{2}\right)$ = (0, 6) **[1]**

13. Let polynomial p(x) = $ax^2 + bx + c$

    Product of the zeros = c/a

    P(x) = $(x^2 + 5x - 6)$

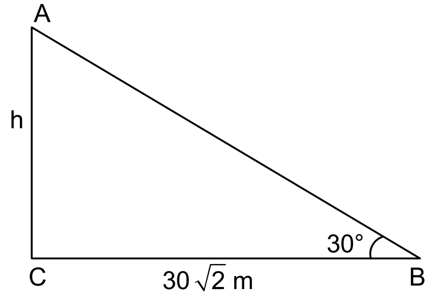
    Product of the zeros = -6/1 = -6 **[1]**

14. Ratio of the sides of two similar triangles = 3 : 7

    We know that if the ratio of the sides of two similar triangle is a : b, then the ratio of their areas is $a^2 : b^2$.

    Ratio of the areas of the given similar triangles = $(3/7)^2$ = 9 : 49 **[1]**

15.



In triangle ABC,

$\tan 30° = \frac{h}{30\sqrt{2}}$

$\frac{1}{\sqrt{3}} = \frac{h}{30\sqrt{2}}$

$h = \frac{30\sqrt{2}}{\sqrt{3}}$

$h = 10\sqrt{6}$ m **[1]**

16. H.C.F. × L.C.M. = Product of two numbers

L.C.M. = $\frac{x}{p}$ **[1]**

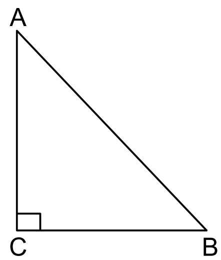
17. $\left(\frac{3}{5} + \frac{2}{3}\right) = \frac{19}{3 \times 5}$

As the denominator is in the form of $3^m \times 5^n$, So the fraction is non-terminating. **[1]**

18. Number of zeros = Number of times the graph intersects or touches the x-axis

$\therefore$ Number of zeros = 0 **[1]**

19.



15 cot A = 8

cot A = 8/15 = AC/BC

Thus, in triangle ABC, AB = 17k, BC = 15k and AC = 8k.

So, sin B = AC/AB = 8/17 **[1]**

20. b + 90° + 35° = 180°

b = 55° **[1]**

OR

In right triangle OPQ,

$OP^2 + PQ^2 = OQ^2$

$OQ^2 = 9^2 + 12^2$

$OQ^2 = 81 + 144$

$OQ^2 = 225$

OQ = 15 cm **[½]**

So, RQ = OQ – OR = 15 – 9 = 6 cm **[½]**

SECTION – B

21. $\frac{987}{10,500} = \frac{3 \times 7 \times 47}{3 \times 7 \times 500} = \frac{47}{500}$ **[1]**

Factors of 500 = 2 × 2 × 5 × 5 × 5

$= 2^2 \times 5^3$ **[½]**

$\therefore$ Prime factors of 500 are in the form of $2^m \times 5^n$.

Therefore, $\frac{987}{10,500}$ is a terminating decimal. **[½]**

OR

The time after which the bells will ring together is the L.C.M. of 2, 3 and 5 seconds, i.e. 30 seconds. **[1]**
The number of times they will toll together in one hour = (3600/30) = 120.
Thus, they will toll together 120 times in an hour. **[1]**

22. The given system of equations will have infinite number of solutions, if $\frac{2m-1}{3}=\frac{3}{n-1}=\frac{-5}{-2}$ **[1]**

$\Rightarrow \frac{2m-1}{3}=\frac{3}{n-1}=\frac{5}{2}$

$\Rightarrow \frac{2m-1}{3}=\frac{5}{2}$ and $\frac{3}{n-1}=\frac{5}{2}$

$\Rightarrow$ 4m – 2 = 15 and 6 = 5n – 5

$\Rightarrow$ 4m = 17 and 5n = 11

$\Rightarrow$ m = $\frac{17}{4}$ **[½]**

And n = $\frac{11}{5}$ **[½]**

Hence, the given system of equations will have infinite number of solutions if m = $\frac{17}{4}$ and n = $\frac{11}{5}$.

OR

x + 3y = 6 …(i)
2x – 3y = 12 …(ii)
Eqn (i) × 2 – Eqn(ii); **[½]**
2x + 6y – 2x + 3y = 12 - 12
9y = 0
y = 0 **[1]**
So, x + 3(0) = 6
x = 6 **[½]**
Solution is x = 6 and y = 0 .

23. It is given that on dividing 398 by the required number, the remainder comes out to be 7. This means that 398 – 7 = 391 is exactly divisible by the required number. In other words, the required number is a factor of 391. **[½]**
Similarly, the required positive integer is a factor of 436 – 11 = 425 and 542 – 15 = 527. **[½]**
Clearly, the required number is the HCF of 391, 425 and 527.
The prime factors of 391, 425 and 527 are as follows:
391 = 17 X 23
425 = 5 X 5 X 17
527 = 17 X 31 **[½]**
$\therefore$ HCF = 17
Hence, the required number is 17. **[½]**

24.

Concentration of SO_2	f_i	x_i	f_ix_i
0.00 - 0.04	4	0.02	0.08
0.04 - 0.08	9	0.06	0.54
0.08 - 0.12	9	0.10	0.90
0.12 - 0.16	2	0.14	0.28
0.16 - 0.20	4	0.18	0.72
0.20 - 0.24	2	0.22	0.44
	$\sum f_i = 30$		$\sum f_ix_i = 2.96$

[1]

Mean $\left(\bar{x}\right)=\frac{\sum f_ix_i}{\sum f_i}=\frac{2.96}{30}$ **[½]**

$\bar{x}$ = 0.0987 ppm **[½]**

25. Total outcomes = 6 × 6 = 36 **[½]**

Favourable outcomes = (5, 6), (6, 5) **[½]**

Probability = $\frac{2}{36} = \frac{1}{18}$ **[1]**

26. Total number of cards = 52

Total number of kings = 4

Total number of red cards = 26

Total number of red king cards = 2

So, total number of cards which are neither king nor red = 52 – 26 – 2 = 24 **[1]**

(As 26 red cards contain 2 red kings and there are 2 black kings also)

Thus, number of favourable cases = 24

Hence, required probability = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} = \frac{24}{52} = \frac{6}{13}$ **[1]**

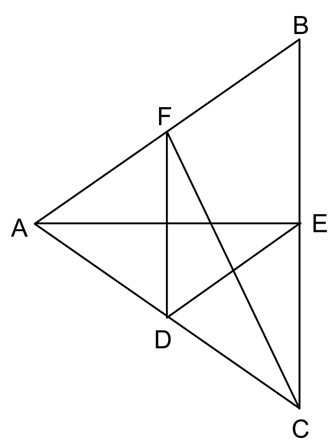
SECTION – C

27. Since AE is the median,

So, BE = $\frac{1}{2}$BC ....(i) **[½]**

Since FD || BC,

So, $\frac{AD}{AC} = \frac{AF}{AB}$ (By BPT) ....(ii) **[½]**



Since ED || AB,

So, $\frac{AD}{AC} = \frac{EB}{BC} = \frac{1}{2}$ [By BPT and from (i)] ....(iii) **[½]**

From equations (ii) and (iii), we get

So, $\frac{AF}{AB} = \frac{1}{2}$ **[½]**

AF = FB

Since F is the midpoint of AB, so CF is the median drawn from C to AB. **[1]**

28. Let AB be the building and CD be the tower such that $\angle$BDE = 30°, $\angle$BCA = 60° and AB = 60 m.

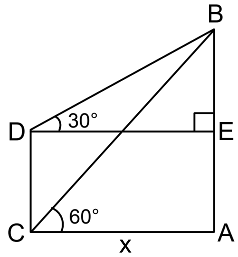
Let CA = DE = x metres

From right ΔCAB, we have

$\frac{CA}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x}{60} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ x = $\left(60 \times \frac{1}{\sqrt{3}}\right) = 20\sqrt{3}$



[1]

$\Rightarrow$ CA = DE = 20 $\sqrt{3}$ m …. (i) [1]

From right Δ BED, we have

$\frac{BE}{DE} = \tan 30^{o} = \frac{1}{\sqrt{3}} \Rightarrow \frac{BE}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$ [using (i)]

$\Rightarrow$ $BE = \left(20\sqrt{3} \times \frac{1}{\sqrt{3}}\right) = 20$ m

$\therefore$ CD = AE = (AB – BE) = (60 – 20) m = 40 m

Hence, the height of the tower is 40 m. [1]

29. Here, three coins are tossed. Therefore, sample space (S) has 2^3 = 8 elements

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore$ n(S) = 8

(i) Let E be the event of getting two heads.

$\therefore$ E = {HHT, HTH, THH}

$\therefore$ n(E) = 3

$\therefore$ P(getting two heads) = $\frac{n(E)}{n(S)} = \frac{3}{8}$ [1]

(ii) Let E be the event of getting at least two heads, i.e. the number of heads is at least 2, which is 2 or 3.

$\therefore$ E = {HHT, HTH, THH, HHH}

$\therefore$ n(E) = 4

$\therefore$ P(at least two heads) = $\frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$ [1]

(iii) Let E be the event of getting at the most two heads, i.e. the number of heads is at most 2, which is 2, 1 or 0. By 0 heads, we mean we are getting all tails.

$\therefore$ E = {HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore$ n(E) = 7

$\therefore$ P(at the most 2 heads) = $\frac{n(E)}{n(S)} = \frac{7}{8}$ [1]

30. By using distance formula,

AB = $\sqrt{(1+3)^2 + (-3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

BC = $\sqrt{(4-1)^2 + (1+3)^2} = \sqrt{9+16} = \sqrt{25} = 5$

AC = $\sqrt{(4+3)^2 + (1-0)^2} = \sqrt{49+1} = \sqrt{50}$ [1½]

Now, $AB^2 + BC^2$ = 25 + 25

= 50 = $\left(\sqrt{50}\right)^2 = AC^2$ [½]

Also, AB = BC = 5

Hence, Δ ABC is an isosceles right triangle. [1]

31. $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

LHS = $\tan^2 A - \tan^2 B$

$= \dfrac{\sin^2 A}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 B} = \dfrac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$ **[1]**

$= \dfrac{\sin^2 A\left(1 - \sin^2 B\right) - \sin^2 B\left(1 - \sin^2 A\right)}{\cos^2 A \cos^2 B}$ $\left[\because \cos^2\theta = 1 - \sin^2\theta\right]$ **[1]**

$= \dfrac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B}$ **[½]**

$= \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$ **[½]**

OR

$45° + \theta = 90° - (45° - \theta)$ **[½]**

$60° + \theta = 90° - (30° - \theta)$ **[½]**

$= \dfrac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° + \theta)}$

$= \dfrac{\cos^2(45° + \theta) + \sin^2(45° + \theta)}{\tan(60° + \theta)\cot(60° + \theta)}$ **[1]**

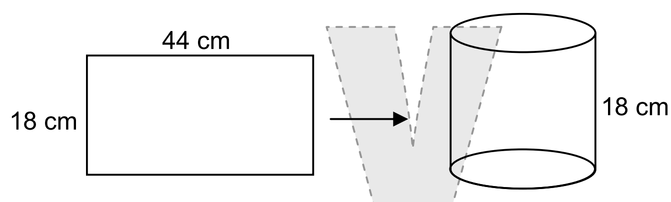
$= \dfrac{1}{1}$

$= 1$ **[1]**

32. The sheet of paper is rolled along the length.

Therefore, height of the cylinder = 18 cm

Circumference of the base of the cylinder = 44 cm



Let 'r' be the radius of the base of the cylinder.

$\therefore \quad 2 \times \dfrac{22}{7} \times r = 44$ cm **[1½]**

$\Rightarrow$ r = 7 cm

Also, height = 18 cm

$\therefore$ Volume = $\pi r^2 h = \dfrac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 18 \text{ cm} = 2772 \text{ cm}^3$ **[1½]**

OR

Height of the container = 16 cm = 0.16 m

Smaller radius (r) = 8 cm = 0.08 m

Larger radius (R) = 20 cm = 0.2 m

Volume of the container (a frustum) = $\dfrac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \dfrac{\pi}{3} \times 0.16 \times (0.04 + 0.0064 + 0.016)$

$= \frac{\pi}{3} \times 0.16 \times 0.0624 \text{ m}^3$ **[1]**

We know, 1 m^3 = 1000 ℓ

$\therefore$ Volume of the container = $\frac{\pi}{3}$ x 0.16 x 0.0624 m^3 = $\frac{\pi}{3}$ x 0.16 x 0.0624 x 1000 ℓ

$= \frac{\pi(0.16 \times 62.4)}{3} \ell$ **[1]**

Now, cost of 1 ℓ milk = Rs. 42

$\therefore$ Total cost = Rs. 439.3 **[1]**

33. $\frac{1}{2x-3} + \frac{1}{x-5} = 1$

$\frac{2x-3+x-5}{(2x-3)(x-5)} = 1$

$3x - 8 = 2x^2 - 10x - 3x + 15$

$\Rightarrow 2x^2 - 13x - 3x + 15 + 8 = 0$

$2x^2 - 16x + 23 = 0$ **[1]**

By using the formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{16 \pm \sqrt{256 - 184}}{4}$

$= \frac{16 \pm \sqrt{72}}{4} = \frac{16 \pm 6\sqrt{2}}{4}$

$x = \frac{2\left(8 \pm 3\sqrt{2}\right)}{4} = \frac{8 \pm 3\sqrt{2}}{2}$ **[1½]**

Hence, the roots are $\frac{8+3\sqrt{2}}{2}$ and $\frac{8-3\sqrt{2}}{2}$. **[½]**

34.

$$
\begin{array}{r|l}
 & 4x^2 - 2x + 7 \\
2x^2 + 3x - 5 & 8x^4 + 8x^3 - 12x^2 + 21x - 30 \\
 & \pm 8x^4 \pm 12x^3 \mp 20x^2 \\
\hline
 & -4x^3 + 8x^2 + 21x \\
 & \mp 4x^3 \mp 6x^2 \pm 10x \\
\hline
 & 14x^2 + 11x - 30 \\
 & \pm 14x^2 \pm 21x \mp 35 \\
\hline
 & -10x + 5
\end{array}
$$

[1]

$q(x) = 4x^2 - 2x + 7$

$r(x) = -10x + 5$

$p(x) = q(x).g(x) + r(x)$

$8x^4 + 8x^3 - 12x^2 + 21x - 30 = (4x^2 - 2x + 7)(2x^2 + 3x - 5) -10x + 5$ **[1]**

RHS = $8x^4 + 12x^3 - 20x^2 - 4x^3 - 6x^2 + 10x + 14x^2 + 21x - 35 - 10x + 5$

$= 8x^4 + 8x^3 - 12x^2 + 21x - 30$

= LHS **[1]**

OR

$7x - x^2 - 6 = - x^2 + 7x - 6$

Now, $-x^2 + 7x - 6 = -x^2 + 6x + x - 6$

$= -x(x - 6) + 1(x - 6)$

$= (x - 6)(- x + 1)$

Hence, the zeros are

$x - 6 = 0 \Rightarrow x = 6$

$-x + 1 = 0 \Rightarrow x = 1$

Sum of zeros = 6 + 1 = 7 **[1]**

Sum of zeros from the polynomial = $\frac{-b}{a} = \frac{-7}{-1} = 7$

Hence, verified. **[1]**

Product of zeros = 6 × 1 = 6

Product of zeros from the polynomial = $\frac{c}{a} = \frac{-6}{-1} = 6$

Hence, verified. **[1]**

SECTION – D

35. Steps of Construction:

Step I: With O as a centre and radius equal to 4 cm, a circle is drawn.

Step II: The diameter of the circle is extended on the both sides and an arc is made to cut it at 6 cm.

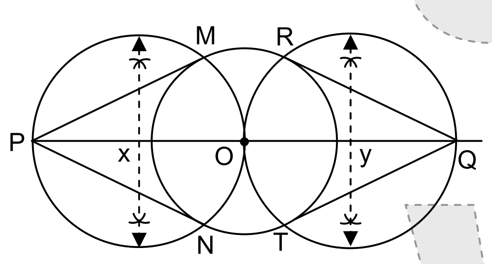
Step III: Perpendicular bisector of OP and OQ is drawn and x and y be its mid-point.

Step IV: With O as a centre and OX be its radius, a circle is drawn which intersects the previous circle at M and N.

Step V: Step IV is repeated with O as centre and Oy as radius and it intersects the circle at R and T.

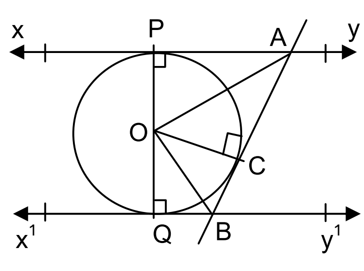
Step VI: PM and PN and, QR and QT are joined.

Thus, PM and PN are tangents to the circle from P, and QR and QT are tangents to the circle from point Q



[1 + 3]

OR



We need to prove that ∠AOB = 90°.

Now: In ΔAOC and ΔAOP

OA = OA (hypotenuse)

OP = OC (radii)

∠ACO = ∠APO (right angle)

∴ ΔAOC ≅ ΔAOP (By RHS congruency)

By CPCT: ∠AOC = ∠AOP ... (1) **[1]**

Similarly, in ΔBOC & ΔBOQ:

OC = OQ

OB = OB

$\angle$BCO = $\angle$BQO = 90°

By RHS congruency: ΔBOC $\cong$ ΔBOQ

By CPCT: $\angle$BOC = $\angle$BOQ ... (2) **[1]**

PQ is a straight line.

$\therefore$ $\angle$AOP + $\angle$AOC + $\angle$BOC + $\angle$BOQ = 180°

From equations (1) and (2): 2($\angle$AOC + $\angle$BOC) = 180°

$\angle$AOB = 180°/2

$\therefore$ $\angle$AOB = 90° **[2]**

36.

Literacy rate (in %age)	Number of cities (f_i)	x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	f_iu_i
45-55	3	50	-20	-2	-6
55-65	10	60	-10	-1	-10
65-75	11	70	0	0	0
75-85	8	80	10	1	8
85-95	3	90	20	2	6
Total	35				-2

[2]

From the above table, $\sum f_i = 35$

$\sum f_iu_i = -2$

Using step deviation method,

Mean $(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$ **[½]**

= 70 + $\left(\frac{-2}{35}\right) \times 10$

= 70 - $\frac{20}{35}$ **[½]**

= $\frac{490 - 4}{7}$ **[½]**

= $\frac{486}{7}$

= 69.43 **[½]**

$\therefore$ The mean literacy rate is 69.43%.

OR

Here, the class size varies, and the x's are large.

Let us still apply the step-deviation method with a = 200 and h = 20. **[½]**

Then, we obtain the data as in the table below.

Number of wickets taken	Number of bowlers (f_i)	x_i	$d_i = x_i - 200$	$u_i = \frac{d_i}{20}$	u_if_i
20-60	7	40	-160	-8	-56
60-100	5	80	-120	-6	-30
100-150	16	125	-75	-3.75	-60
150-250	12	200	0	0	0
250-350	2	300	100	5	10
350-450	3	400	200	10	30
Total	45				-106

[½ + ½ + ½ + ½]

So, $\bar{u} = \frac{-106}{45}$

Therefore, $\overline{x} = 200 + 20\left(\frac{-106}{45}\right)$ = 200 - 47.11 = 152.89 **[1½]**

This tells us that, on an average, the number of wickets taken by these 45 bowlers in one day cricket match is 152.89.

37. Volume of the hemisphere = $\frac{1}{2}\times\frac{4}{3}\pi r^3 = \frac{1}{2}\times\frac{4}{3}\times\frac{22}{7}\times\left(\frac{3}{4}m\right)^3$

$= \frac{1}{2}\times\frac{4}{3}\times\frac{22}{7}\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\ m^3$ **[1]**

$= \frac{99}{112}\ m^3$

$= \frac{99000000}{112}\ cm^3$ **[1]**

Volume to be emptied = $\frac{1}{2}\times\frac{99000000}{112}\ cm^3$ **[1]**

Also, 1 litre = 1000 cm^3

Volume to be emptied = $\frac{1}{2}\times\frac{99000000}{112}\times\frac{1}{1000}$ litres

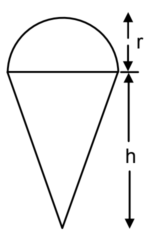
$= \frac{49500}{112}$ litres^3 litres are emptied in 1 minute.

$\therefore$ $\frac{49500}{112}$ litres will be emptied in $\frac{1}{3}\times\frac{49500}{112}$ minutes.

$= \frac{1}{3}\times\frac{49500}{112\times 60}$ hours

$= 2\ \frac{51}{112}$ hours **[1]**

OR



Ice cream cone

Volume of ice cream in the container = $\pi r^2 h$

$= \pi \times 6 \times 6 \times 15\ cm^3$

$= 540\pi\ cm^3$

Volume of ice cream in one cone = Volume of hemispherical top + Volume of conical part

$= \frac{2}{3}\pi\times r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi\times(3\ cm)^3 + \frac{1}{3}\pi\times 3\ cm\times 3\ cm\times 12\ cm$

$= 18\pi\ cm^3 + 36\pi\ cm^3$

$= 54\pi\ cm^3$ **[2]**

Number of cones = $\frac{\text{Total volume of ice cream}}{\text{Volume of ice cream in 1 cone}} = \frac{540\pi\ cm^3}{54\pi\ cm^3} = 10$ **[2]**

38. $a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$a_3 = a + 2d$ **[½]**

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$ **[½]**

$a = 3 - 4d$ ....(i)

Also, it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

from equation (i),

$(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$

$(3 - 2d) \times (3 + 2d) = 8$

$9 - 4d^2 = 8$

$4d^2 = 9 - 8 = 1$

$d^2 = \frac{1}{4}$

$d = \pm\frac{1}{2}$ **[1]**

$d = \frac{1}{2}$ or $\frac{1}{2}$

From equation (i),

$\left(\text{When d is } \frac{1}{2}\right)$

$a = 3 - 4d$

$a = 3 - 4\left(\frac{1}{2}\right)$

$= 3 - 2 = 1$ **[½]**

$\left(\text{When d is } -\frac{1}{2}\right)$

$a = 3 - 4\left(-\frac{1}{2}\right)$

$a = 3 + 2 = 5$ **[½]**

$S_a = \frac{n}{2}\ [2a(n - 1)d]$

$\left(\text{When a is 1 and d is } \frac{1}{2}\right)$

$S_a = \frac{16}{2}\left[2(1) + (16 - 1)\left(\frac{1}{2}\right)\right]$

$= 8\left[2 + \frac{15}{2}\right]$

$= 4(19) = 76$ **[½]**

$\left(\text{When a is 5 and d is } -\frac{1}{2}\right)$

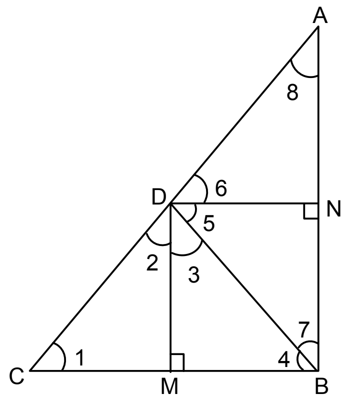
$S_a = \frac{16}{2}\left[2(5) + (16 - 1)\left(-\frac{1}{2}\right)\right]$

$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$

$= 8\left(\frac{5}{2}\right)$

$= 20$ **[½]**

39. Construction: Join DB.



As we know that DN || CB, DM || AB and $\angle B = 90°$,

$\therefore$ DMBN is a rectangle.

$\therefore$ DN = MB and DM = NB **[½]**

$\angle CDB = 90°$

$\Rightarrow \angle 2 + \angle 3 = 90°$ ... (i) **[½]**

In ΔCDM,

$\angle 1 + \angle 2 + \angle DMC = 180°$

$\Rightarrow \angle 1 + \angle 2 = 90°$ ... (ii) **[½]**

In ΔDMB,

$\angle 3 + \angle DMB + \angle 4 = 180°$

$\Rightarrow \angle 3 + \angle 4 = 90°$ ... (iii) **[½]**

From equations (i) and (ii), $\angle 1 = \angle 3$

From equations (i) and (iii), $\angle 2 = \angle 4$

In ΔBDM and ΔDCM,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$ (Proved above)

$\therefore \Delta BDM \sim \Delta DCM$ (By AA similarity criterion) **[1]**

$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ [$\because$ BM = DN]

$\Rightarrow DM^2 = DN \times MC$ **[1]**

40. (i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ (Common) **[½]**

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is $\angle PCA = \angle PBD$ equal to the opposite interior angle) **[1]**

$\therefore \Delta PAC \sim \Delta PDB$ [By AA similarity criterion] **[½]**

(ii) We know that the corresponding sides of similar triangles are proportional. **[½]**

$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$ **[1]**

$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$

$\therefore$ PA.PB = PC.PD **[½]**

MATHEMATICS (BASIC)

(SOLUTIONS)

SECTION – A

1. **Answer:** (4) **[1]**
2. **Answer:** (1) **[1]**
3. **Answer:** (4) **[1]**
4. **Answer:** (1) **[1]**
5. **Answer:** (1) **[1]**
6. **Answer:** (2) **[1]**
7. **Answer:** (1) **[1]**
8. **Answer:** (1) **[1]**
9. **Answer:** (3) **[1]**
10. **Answer:** (1) **[1]**

11. A $(x_1, y_1) = (0,5)$, B$(x_2, y_2) = (5,0)$

$m_1 : m_2 = 3 : 2$ (divides internally)

As we know,

By section formula $(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$ **[½]**

$(x, y) = \left(\frac{3 \times 5 + 2 \times 0}{3 + 2}, \frac{3 \times 0 + 2 \times 5}{3 + 2}\right) = (3, 2)$ **[½]**

12. The equations 4x + 2y + 6 = 0 and 2x + ky + 3 = 0 will be parallel if

$\frac{4}{2} = \frac{2}{k} = \frac{6}{3}$

k = 1 **[1]**

OR

$x^2 - 6x + k = 0$ has equal roots

$D = b^2 - 4ac = 0$

$(-6)^2 - 4k = 0$

k = 9 **[1]**

13. sin(A + B) = sinA cosB + cosA sinB **[½]**

sin60° cos30° + sin30° cos60° = sin(60° + 30°) = sin90° = 1 **[½]**

14. cos0° . cos30° . cos45° . cos60° . cos90°

= cos0° cos30° cos45° cos60° × 0

= 0 **[1]**

15. The sides of two similar triangles in the ratio 1 : 3

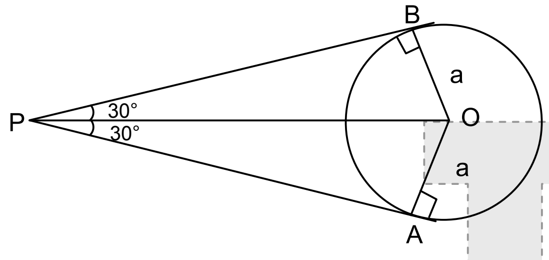
The ratio of the areas of these triangles = $(1 : 3)^2$ = 1 : 9 **[1]**

16. Two triangular plots are equiangular, so these triangular are similar triangles.

The areas of these triangles in the ratio 5 : 20.

Ratio of their sides = Ratio of their perimeters = $\sqrt{\frac{5}{20}}$ = 1 : 2 **[1]**

17.



Given: $\angle BPA = 60°$

OB = OA = a [Radii]

PA = PB [Length of tangents are equal]

Δ OP = OP

$\therefore$ ΔPBO and ΔPAO are congruent.

$\therefore$ $\angle BPO = \angle OPA = \frac{60^o}{2} = 30^o$ **[½]**

We know that a tangent is always perpendicular to the radius at the point of contact.

So, in Δ PBO,

$\sin 30° = \frac{a}{OP}$

OP = 2a **[½]**

18. Suppose $\Delta ABC \sim \Delta DEF$

$\therefore$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$ (Given)

Now, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB + BC + AC}{DE + EF + DF}$

$= \frac{\frac{2}{3}DE + \frac{2}{3}EF + \frac{2}{3}DF}{DE + EF + DF}$

$= \frac{\frac{2}{3}(DE + EF + DF)}{(DE + EF + DF)}$

$= \frac{2}{3}$ **[1]**

19. tan(55° - θ) - cot(35° + θ) = tan{90° - (35° + θ)} - cot(35° + θ)

= cot(35° + θ) - cot(35° + θ) = 0 **[1]**

OR

(1/cot2θ) - sec2θ = tan2θ - sec2θ = $\left(\frac{\sin 2\theta}{\cos 2\theta} - \frac{1}{\cos 2\theta}\right) = \frac{\sin 2\theta - 1}{\cos 2\theta}$ **[1]**

20. We have,

$\sqrt{x^2 + y^2} = \sqrt{(x+5)^2 + (y-6)^2}$

$x^2 + y^2 = x^2 + 25 + 10x + y^2 + 36 - 12y$

10x - 12y + 61 = 0 **[1]**

SECTION – B

21. Number of cards left = 52 - 3 = 49

Number of cards of spades left = 13 - 3 = 10

Number of black cards left = 13 + 10 = 23

(Cards of spades are of black colour)

Total number of cards = 49 **[½]**

Total number of black cards (favourable) = 23 **[½]**

$\therefore$ Required probability = $\frac{23}{49}$ **[1]**

22. 2x = 3y - 6

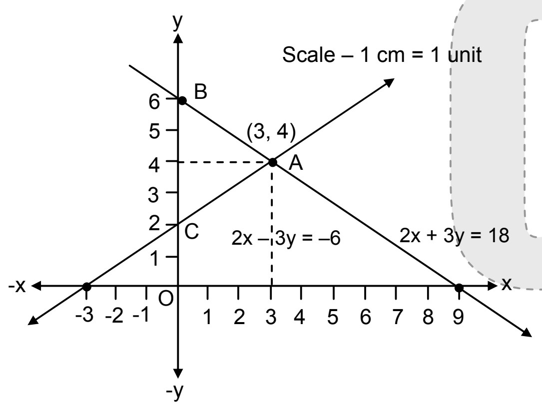
$x = \frac{3y-6}{2}$ **[½]**

x	-3	0	3
y	0	2	4

2x = 18 - 3y

$x = \frac{18-3y}{2}$

x	9	6	3
y	0	2	4



[½ mark for each correct graphing of an equation]

Both lines are intersect at point A (3,4).

Area of the region bounded by these two lines and y-axis = Area of Δ ABC = $\frac{1}{2}\times 4\times 3$ = 6 sq. units **[1]**

23. 2x + 3y = 7

8x + 12y = 28 ....(i)

2ax + (a + b)y = 28 ....(ii)

Equations (i) and (ii) have infinitely many solutions, so both the equations are same.

2a = 8 , (a + b) = 12 **[1]**

a = 4 , b = 12 - 4 = 8 **[1]**

24. It is given that $\frac{2}{3}$, k, $\frac{5}{8}k$ are in AP.

By the definition of AP,

$k - \frac{2}{3} = \frac{5}{8}k - k$ [$t_2 - t_1 = t_3 - t_2$] **[1]**

$\Rightarrow$ $k + k - \frac{5}{8}k = \frac{2}{3} \Rightarrow \frac{11}{8}k = \frac{2}{3}$

$\therefore$ $k = \frac{2}{3}\times\frac{8}{11} = \frac{16}{33}$ **[1]**

OR

Let 'a' be the first term and 'd' be the common difference .

So, 5(a + 4d) = 12(a +11d) **[1]**

$5a + 20d = 12a + 132d$

$7a + 112d = 0$

$7(a + 16d) = 0$

$7(a_{17}) = 0$

17^{th} term = 0 **[1]**

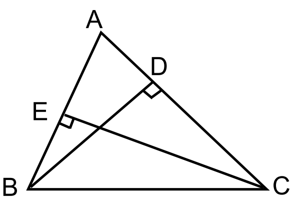
25. Total number of tickets = 40

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35 and 40.

Numbers which are not multiples of 3, but are multiples of 5 are 5, 10, 20, 25, 35 and 40. **[1]**

P(a number which is a multiple of 5) = $\frac{6}{40} = \frac{3}{20}$ **[1]**

26. In triangles BDA and CEA,



$\angle BDA = \angle CEA = 90^o$ [Given]

$\angle BAD = \angle CAE$ [Common] **[½]**

So, by AA criterion of similarity, we have

$\Delta BDA \sim \Delta CEA$ **[½]**

Consequently, $\frac{AB}{AC} = \frac{AD}{AE}$ [If Δs are similar, the ratio of their corresponding sides is same.]

$\Rightarrow \quad AB \times AE = AC \times AD$ **[1]**

OR

AP = AS … (1)

DR = DS … (2)

RC = CQ … (3)

BP = QB … (4) **[½]**

AP + DR + RC + BP = AS + DS + CQ + QB **[½]**

AP + PB + DR + RC = AS + SD + CQ + QB

AB + DC = AD + CB **[½]**

As AB = AD,

CD = BC **[½]**

SECTION – C

27. Total number of possible outcomes when two dice are rolled = 36

(1) Favorable Cases = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)

P(the same number on both the dice) = $\frac{6}{36} = \frac{1}{6}$ **[1]**

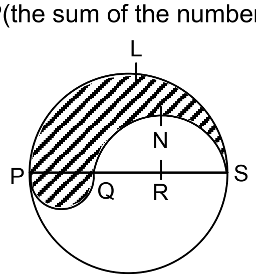
(2) Favorable Cases = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (3, 1), (4, 1), (5, 1) and (6, 1)

P(1 on any of the dice) = $\frac{11}{36}$ **[1]**

(3) Favorable Cases = (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1)

P(the sum of the numbers on both the dice equal to 7) = $\frac{6}{36} = \frac{1}{6}$ **[1]**

28.



Since PS = 6 cm and PQ = QR = RS,

$\therefore \quad PQ = \frac{1}{3} PS = \frac{1}{3} \times 6 = 2$ cm **[½]**

Area of semicircle with diameter PQ = $\frac{\theta}{360^o} \times \pi r^2$ **[½]**

$= \frac{180^o}{360^o} \times \frac{22}{7} \times 1 \times 1 \text{ cm}^2 = \frac{11}{7} \text{cm}^2$

Length of QS = QR + RS = 2 + 2 = 4 cm

Area of semicircle with diameter SQ = $\frac{\theta}{360^o} \times \pi r^2$

$= \frac{180^o}{360^o} \times \frac{22}{7} \times 2 \times 2 \text{ cm}^2 = \frac{44}{7} \text{cm}^2$ **[½]**

Area of semicircle PLS = $\frac{\theta}{360} \times \pi r^2$

$= \frac{180^o}{360^o} \times \frac{22}{7} \times 3 \times 3 \text{ cm}^2 = \frac{99}{7} \text{cm}^2$ **[½]**

Area of the shaded region = Area of semicircle PLS - Area of sector RQNS + Area of the semicircle with diameter PQ **[½]**

$= \frac{99}{7} - \frac{44}{7} + \frac{11}{7} \text{cm}^2$

$= \frac{99 - 44 + 11}{7} = \frac{11(9 - 4 + 1)}{7} \text{ cm}^2$

$= \frac{11 \times 6}{7} = \frac{66}{7} \text{ cm}^2$ **[½]**

29. Let us assume that $\sqrt{5}$ is rational.

$\Rightarrow \quad \sqrt{5}$ can be written in the form of $\frac{a}{b}$; $b \neq 0$.

$\therefore \quad \frac{a}{b} = \sqrt{5}$

$\frac{a}{b} = \sqrt{5}$; where, a and b are co-prime.

$a = b\sqrt{5}$ **[1]**

Squaring both the sides, we get:

$a^2 = 5b^2$ ... (i)

$\therefore \quad$ 5 divides a^2.

$\Rightarrow \quad$ 5 divides a.

$\therefore \quad$ a = 5c for some integer c.

Substituting for a = 5c in equation (i), we get

$(5c)^2 = 5b^2$

$5c^2 = b^2$ **[1]**

$\therefore \quad$ 5 divides b^2.

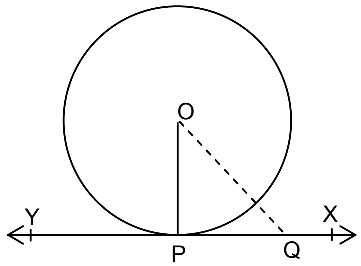
$\Rightarrow \quad$ 5 divides b, which is not possible as a and b are co-prime.

HCF (a, b) = 1

$\therefore \quad$ Our assumption is wrong.

So, $\sqrt{5}$ is irrational. **[1]**

30.



[1]

We are given a circle with centre O and a tangent XY to the circle at a point P. We need to prove that OP is perpendicular to XY

Take a point Q on XY other than P and join OQ.

The point Q must lie outside the circle.

(Note that if Q lies inside the circle, XY will become a secant and not a tangent to the circle.)

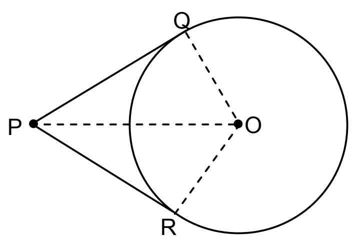
Therefore, OQ is longer than the radius OP of the circle.

That is, OQ > OP. **[1]**

Since this happens for every point on the line XY except the point P, OP is the shortest of all the distances of the point O to the points of XY.

So, OP is perpendicular to XY. **[1]**

OR



[1]

We are given a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P.

We are required to prove that PQ = PR.

For this, we join OP, OQ and OR.

Then $\angle$OQP and $\angle$ORP are right angles because these are angles between the radii and tangents.

Now in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

OP = OP (Common)

Therefore, ΔOQP $\cong$ ΔORP (RHS) **[1]**

This gives PQ = PR (CPCT) **[1]**

31. $S_n = 2n + 3n^2$ ... (1)

Changing n to n - 1, we get

$S_{n-1} = 2(n - 1) + 3(n - 1)^2$ ... (2) **[½]**

Now, $t_n = S_n - S_{n-1}$ **[1]**

$t_n = (2n + 3n^2) - [2(n - 1) + 3(n - 1)^2]$

$t_n = 2n + 3n^2 - 2(n - 1) - 3(n - 1)^2$

$t_n = 2[(n - (n - 1)] + 3[n^2 - (n - 1)^2]$

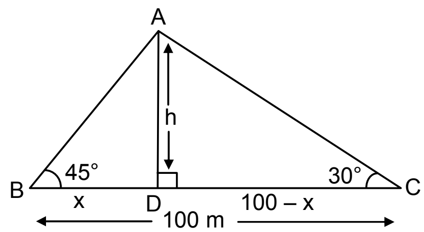
$t_n = 2[(n - n + 1)] + 3[n + (n - 1)][n - (n - 1)]$

$t_n = 2 + 3(2n - 1)$

$t_n = 6n - 1$ **[1]**

$\therefore$ Putting n = r, $t_r = 6r - 1$ **[½]**

32. AD is the lighthouse.



[1 mark for the correct figure]

In right ΔADB,

$\Rightarrow \frac{h}{x} = \tan 45^o$

So, h = x ... (i) **[½]**

Now, in right ΔADC,

$\frac{h}{100 - x} = \tan 30^o$ ... (ii) **[1]**

Solving for h and x:

$\Rightarrow \quad \frac{h}{100 - x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h = 100 - x$

$\Rightarrow \quad \sqrt{3}x$ = 100 - x [Using(i)] **[½]**

$\Rightarrow \quad (\sqrt{3} + 1)\, x = 100 \text{ m} \Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{2} = 50(\sqrt{3}-1)\text{m}$ **[1]**

$\therefore \quad$ h = height of the lighthouse = $50(\sqrt{3} - 1)$ m

33. Let P(x, y), A(a + b, b - a) and B(a - b, a + b) be the given points.

Since AP = BP,

$\Rightarrow \quad AP^2 = BP^2$ **[1]**

$\Rightarrow \quad (x - a - b)^2 + (y - b + a)^2 = (x - a + b)^2 + (y - a - b)^2$

$\Rightarrow \quad (x - a - b)^2 - (x - a + b)^2 = (y - a - b)^2 - (y - b + a)^2$

$\Rightarrow \quad (x - a - b + x - a + b)(x - a - b - x + a - b) = (y - a - b + y - b + a)(y - a - b - y + b - a)$ **[1]**

$\Rightarrow \quad (2x - 2a)(-2b) = (2y - 2b)(-2a)$

$\Rightarrow \quad -4bx + 4ab = -4ay + 4ab$

$\Rightarrow \quad -4bx = -4ay$

$\therefore \quad bx = ay$ **[1]**

OR

Given: - A (0, 3), B (- 2, a) and C (- 1, 4) be the vertices of a right triangle, right-angled at A.

AB = $\sqrt{(0-(-2))^2 + (3-a)^2} = \sqrt{4 + (3-a)^2}$

BC = $\sqrt{(-2+1)^2 + (a-4)^2} = \sqrt{1 + (a-4)^2}$

AC = $\sqrt{(0+1)^2 + (3-4)^2} = \sqrt{2}$ **[1]**

ABC is a right triangle, right-angled at A.

$\therefore$ Using Pythagoras theorem, we get

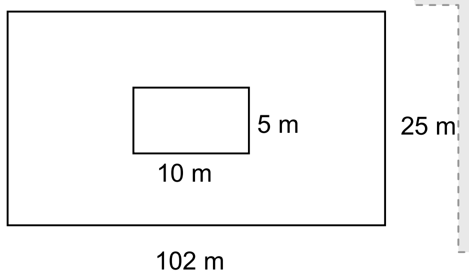
$BC^2 = AB^2 + AC^2$

Or

$1 + (a - 4)^2 = 4 + (3 - a)^2 + 2$ **[1]**

On solving, we get a = 1. **[1]**

34.



Volume of the earth taken out = 10 m × 5 m × 4 m = 200 m^3

Area of the field = 102 m × 25 m = 2550 m^2 **[1]**

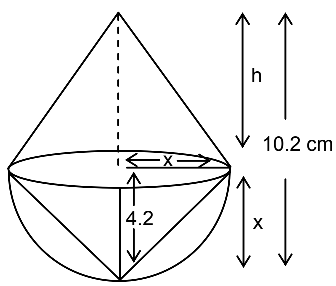
Area of the tank = 10 m × 5 m = 50 m^2

Area of the remaining field = 2550 m^2 - 50 m^2 = 2500 m^2 **[1]**

Level of the field risen = $\frac{200}{2500} m = \frac{2}{25} m$

= $\frac{2}{25} \times 100$ cm = 8 cm **[1]**

OR



r = 4.2 cm

Total height = 10.2 cm

Height of the cone, h = 10.2 cm - 4.2 cm = 6 cm **[½]**

Volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ **[1]**

[$\because$ Volume of hemisphere = $\frac{2}{3}\pi r^3$; volume of cone = $\frac{1}{3}\pi r^2 h$]

= $\frac{\pi r^2}{3}$ [2r + h] **[½]**

= $\frac{22}{7} \times \frac{4.2 \times 4.2}{3}$ × [8.4 + 6]

= $\frac{22}{21} \times \frac{42}{10} \times \frac{42}{10} \times 14.4$

= 22 × 84 × 0.144 = 266.11 cm^3 **[1]**

SECTION – D

35. Since -3 and 3 are zeros of the polynomial $2x^4 - 7x^3 - 13x^2 + 63x - 45$, then $(x + 3)$ and $(x - 3)$ are factors of the polynomial.

$\therefore$ $(x + 3)(x - 3) = x^2 - 9$ will divide the polynomial. **[1]**

$$
\begin{array}{r|l}
 & 2x^2 - 7x + 5 \\
x^2 - 9 & 2x^4 - 7x^3 - 13x^2 + 63x - 45 \\
 & \pm 2x^4 \quad \mp 18x^2 \\ \hline
 & -7x^3 + 5x^2 + 63x - 45 \\
 & \mp 7x^3 \quad \pm 63x \\ \hline
 & 5x^2 - 45 \\
 & \pm 5x^2 \mp 45 \\ \hline
 & 0
\end{array}
$$

[1½]

Factorising $2x^2 - 7x + 5$, we get

$2x^2 - 7x + 5 = 2x^2 - 5x - 2x + 5$

$= x(2x - 5) - 1(2x - 5)$

$= (x - 1)(2x - 5)$ **[1]**

Either $x - 1 = 0$ or $2x - 5 = 0$

$x = 1$ or $x = \frac{5}{2}$ **[½]**

Hence, all the zeros of the polynomial are 3, -3, 1 and $\frac{5}{2}$.

OR

Let the breadth of the rectangular park be x m.

So, its length = (x + 3) m

Therefore, the area of the rectangular park = $x(x + 3)\ m^2$

$= (x^2 + 3x)\ m^2$

Now, base of the isosceles triangle = x m **[1]**

Therefore, its area = $12 \times x \times 12 = 6x\ m^2$ **[1]**

According to our requirements,

$x^2 + 3x = 6x + 4$

i.e., $x^2 - 3x - 4 = 0$

Using the quadratic formula, we get

$$x = \frac{3 \pm \sqrt{9 + 16}}{2}$$

$x = -1, 4$ **[1]**

But $x \neq -1$.

Therefore, x = 4.

So, the breadth of the park is 4 m and its length will be 7 m. **[1]**

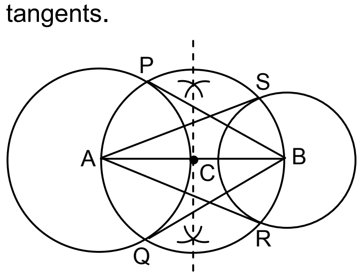
36. The tangents can constructed on the given circles as follows.

Step 1

Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents. **[1 + 1]**



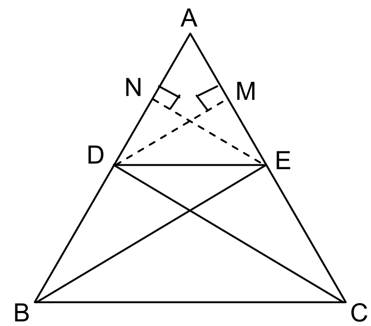
[2]

37. Given: A triangle ABC in which a line parallel to BC intersects other two sides AB and AC at D and E, respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, CD, and draw DM $\perp$ AC and EN $\perp$ AB. **[½]**

Proof: Since EN is perpendicular to AB, EN is the height of triangles ADE and BDE.



$\therefore \quad ar(\Delta ADE) = \frac{1}{2}(\text{base} \times \text{height})$

$= \frac{1}{2}(AD \times EN)$ ... (i) **[½]**

$ar(\Delta BDE) = \frac{1}{2}(\text{base} \times \text{height})$

$= \frac{1}{2}(DB \times EN)$ ... (ii) **[½]**

$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}(AD \times EN)}{\frac{1}{2}(DB \times EN)}$ [Using (i) and (ii)] **[½]**

$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB}$ ... (iii)

Similarly, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}(AE \times DM)}{\frac{1}{2}(EC \times DM)} = \frac{AE}{EC}$ ... (iv)

Note that Δ BDE and Δ DEC are on the same base DE and between the same parallels BC and DE.

$\therefore \quad ar(\Delta BDE) = ar(\Delta DEC)$ ... (v) **[½]**

From (iv) and (v),

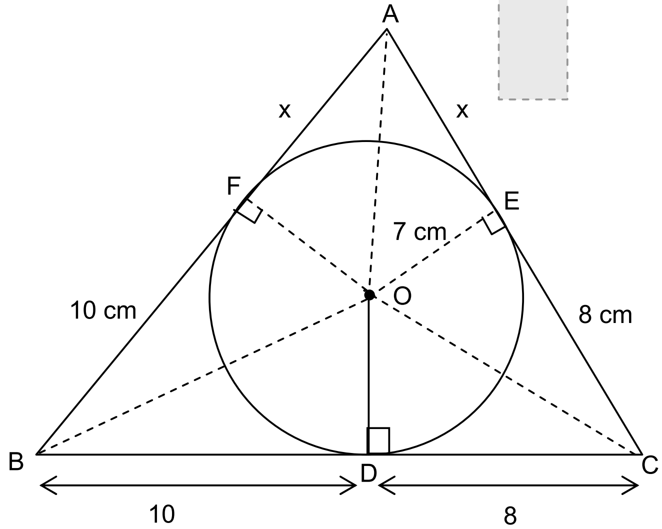
$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC}$ ... (vi) **[½]**

Again, from (iii) and (vi),

$\frac{AD}{DB} = \frac{AE}{EC}$

Hence, $\frac{AD}{DB} = \frac{AE}{EC}$ **[1]**

38.



[Figure mark ½]

CD = 8 cm

CE = 8 cm [Tangents are equal from external point.]

BD = 10 cm

BF = 10 cm

AE = AF = x cm

OD = OE = OF = 7 cm **[½]**

$s = \frac{(x+10)+(10+8)+(8+x)}{2}$

$= \frac{2x+36}{2}$

= (x + 18) cm

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

$= \sqrt{(x+18)(x+18-18)(x+18-8-x)(x+18-10-x)}$

$= \sqrt{(x+18)x(10)(8)}$

$= \sqrt{80x(x+18)}$ **[1]**

Area of Δ OCA = $\frac{1}{2} \times AC \times OE = \frac{1}{2} \times (8 + x) \times 7$

Area of Δ OBC = $\frac{1}{2} \times BC \times OD = \frac{1}{2} \times 18 \times 7 = 63\ cm^2$

Area of Δ OBA = $\frac{1}{2} \times AB \times OF = \frac{1}{2} \times (10 + x) \times 7$

ar(ABC) = ar(OCA) + ar(OBC) + ar(OBA) **[1]**

$\sqrt{80x(x+18)} = \frac{1}{2}(8 + x)7 + 63 + \frac{1}{2}(10 + x)7$

$= \frac{1}{2} \times 7(8 + x + 10 + x) + 63$

$= \frac{7}{2}(18 + 2x) + 63$

= 7(9 + x) + 63

= 7(9 + x + 9)

= 7(18 + x)

$80x(x + 18) = 49(x + 18)^2$

80x = 49(x + 18)

80x = 49x + 882

31x = 882

$x = \frac{882}{31}$

= 28.45 cm

AB = 10 + 28.45 = 38.45 cm

AC = 8 + 28.45 = 36.45 cm **[1]**

39. Let the first pipe take x minutes to fill the cistern.

Then, the second pipe takes (x + 1) minutes to fill the cistern.

Part of the cistern filled by the first pipe in one minute = $\left(\frac{1}{x}\right)^{th}$ part

Part of the cistern filled by the second pipe = $\left(\frac{1}{x+1}\right)^{th}$ part

Two pipes together can fill $\left(\frac{1}{x}+\frac{1}{x+1}\right)=\left(\frac{2x+1}{x(x+1)}\right)^{th}$ part of the cistern in one minute.

$\therefore$ Time taken to fill the cistern = $\frac{1}{\text{part filled in 1 minute}}$

$= \frac{x(x+1)}{2x+1}$ minutes **[1]**

We are given the time taken to fill the cistern equal to $2\frac{8}{11}$ minutes.

Thus, $\frac{x(x+1)}{2x+1}=\frac{30}{11}$ **[½]**

$\Rightarrow$ $11(x^2 + x) = 30(2x + 1)$

$\Rightarrow$ $11x^2 + 11x = 60x + 30$

$\Rightarrow$ $11x^2 - 49x - 30 = 0$

$\Rightarrow$ $11x^2 - 55x + 6x - 30 = 0$ **[1]**

$\Rightarrow$ $11x(x - 5) + 6(x - 5) = 0$

$\Rightarrow$ $(11x + 6)(x - 5) = 0 \Rightarrow x = -\frac{6}{11}, 5$ **[1]**

As $x = -\frac{6}{11}$ is not possible,

Therefore, x = 5 **[½]**

Thus, the pipes take 5 minutes and 6 minutes to fill the cistern.

OR

Let the speed of the stream be x km/h.

Therefore, the speed of the boat upstream = (18 - x) km/h

And, the speed of the boat downstream = (18 + x) km/h

The time taken to go upstream = $\frac{\text{distance}}{\text{speed}}=\frac{24}{18-x}$ hours **[½]**

Similarly, the time taken to go downstream = $\frac{24}{18+x}$ hours **[½]**

According to the question,

$\frac{24}{18-x}-\frac{24}{18+x}=1$

i.e., 24(18 + x) - 24(18 - x) = (18 - x) (18 + x)

i.e., $x^2 + 48x - 324 = 0$ **[1]**

Using the quadratic formula, we get

$x = \frac{-48\pm\sqrt{48^2+1296}}{2}=\frac{-48\pm\sqrt{3600}}{2}$

$= \frac{-48\pm 60}{2}$ = 6 or - 54 **[1]**

Since x is the speed of the stream, it cannot be negative. So, we ignore the root x = - 54.

Therefore, x = 6 gives the speed of the stream as 6 km/h. **[1]**

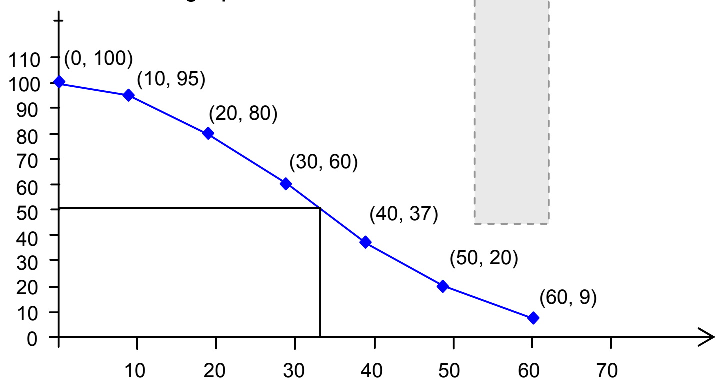
40. Converting into 'more than type' ogive, we get

Class	Frequency
More than 0	100
More than 10	95
More than 20	80

More than 30	60
More than 40	37
More than 50	20
More than 60	9

[2]

Now, draw the graph.



[2]

Median = 34

OR

First, we have to construct the cumulative frequency table as follows:

Class	No. of students(f)	Cum. frequency
0 - 10	5	5
10 - 20	x	5 + x
20 - 30	20	25 + x
30 - 40	15	40 + x
40 - 50	y	40 + x + y
50 - 60	5	45 + x + y
Total	60	

[1]

Now, it is clear that 45 + x + y = 60 or x + y = 15 **[½]**

The median is 28.5 and lies in the class 20 - 30.

Therefore, the median class is 20 - 30. **[½]**

We know the following facts:

$\ell = 20$

n = 60

cf = x + 5

f = 20

h = 10 **[½]**

$$\text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right)_h$$ **[½]**

$$28.5 = 20 + \left(\frac{30 - (x + 5)}{20}\right) \times 10$$

$$= 20 + \left(\frac{30 - x - 5}{20}\right) \times 10$$

$= 20 + \frac{25 - x}{2}$

Now, $\frac{25 - x}{2} = 28.5 - 20 = 8.5$

$25 - x = 8.5 \times 2 = 17$

$x = 25 - 17 = 8$

Now, $x = 8$ **[½]**

$x + y = 15$

$\therefore \quad y = 7$ **[½]**