CHAPTER END TEST
REAL NUMBERS
SOLUTIONS

SECTION – A

1. It states that every composite number can be expressed as product of primes and this factorization is unique, apart from the order in which the prime factors occur. [1]

2. Irrational [1]

3. Let $a = 3 + \sqrt{5}$ and $b = 2 - \sqrt{5}$

   Clearly, a and b are irrational numbers.

   Now, $a + b = 3 + \sqrt{5} + 2 - \sqrt{5} = 5$ (which is a rational number). [½]

   and $a \times b = (3 + \sqrt{5})(2 - \sqrt{5})$

   $= 6 - \sqrt{5} - 5$

   $= 1 - \sqrt{5}$ (which is an irrational number). [½]

4. Let a = p, b = q

   $a = bq + r,\ 0 \le r < b$

   $p = b \times q + p,\ 0 \le p < q$ [1]

5. First find H.C.F. of 86 and 98

   $98 = 86 \times 1 + 12$

   $86 = 12 \times 7 + 2$

   $12 = 2 \times 6 + 0$

   H.C.F. of 98 and 86 is 2

   Now let's find H.C.F. of 86 and 108

   $108 = 86 \times 1 + 22$

   $86 = 22 \times 3 + 20$

   $22 = 20 \times 1 + 2$

   $20 = 2 \times 10 + 0$

   H.C.F. of 86 and 108 is 2.

   H.C.F. of 3 numbers 86, 98, 108 is 2. [1]

6. If q is not of the form $2^n 5^m$, then $\frac{p}{q}$ has decimal expansion which is non–terminating repeating. [1]

7. The last digit of product of first 100 prime numbers would be 0 because it has only one pair of 5×2 in its prime factorization. **[1]**

SECTION – B

8. If the number 4^n were to end with the digit zero, then it would be divisible by 5. That is, the prime factorization of 4^n would contain the prime number 5. This is not possible because $4^n = (2)^{2n}$, so only prime in the factorization of 4^n is 2. So the uniqueness of the fundamental theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with zero. **[2]**

9. LCM (306, 657)

$= \dfrac{306 \times 657}{\text{HCF }(306, 657)}$ **[1]**

$= \dfrac{306 \times 657}{9}$

$= 22338$ **[1]**

10. By applying the Euclid's division lemma, we can find the maximum number of columns in which 616 members (army contingent) and 32 members (army band) can march. HCF of 616 and 32 is equal to maximum number of columns in which 616 and 32 members can march.

Since 616 > 32, we apply the division lemma to 616 and 32, to get:

$616 = 32 \times 19 + 8$ [½]

Since the remainder $r \neq 0$, we apply the division lemma, to get:

$32 = 8 \times 4 + 0$ [½]

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 8.

Therefore, the maximum number of columns in which both 616 members (army contingent) and 32 members (army band) can march is 8. **[1]**

11. Take a = 91, b = 26 [½]

$a = bq + r,\ 0 \leq r < b$ (Division lemma) [½]

$91 = 26 \times 3 + 13$

$26 = 13 \times 2 + 0$ [½]

$\therefore$ H.C.F of 26 and 91 = 13. [½]

12. $7 \times 8 \times 15 + 15 = 15 \times (7 \times 8 \times 1 + 1)$ [½]

As $7 \times 8 \times 15 + 15$ is written as the product of two numbers apart from 1 and number itself. [½]

$\therefore$ It is composite number.

Similarly:

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 7 = 7 \times (6 \times 5 \times 4 \times 3 \times 2 \times 1 + 1)$ [½]

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 7$ is a composite number. [½]

SECTION – C

13. Let the given rational number 0.2323 .... be equal to x.

$x = 0.\overline{23}$ ....(i) [½]

$100x = 23.\overline{23}$ ....(ii) [1]

Subtracting equation (i) from equation (ii)

$100x - x = 23$ [½]

$99x = 23$

$x = \frac{23}{99}$ [1]

14. Let us assume, to the contrary, that $7 - \sqrt{3}$ is rational. [½]

Let $7 - \sqrt{3} = \frac{a}{b}$ where a and b are co–prime integers, $b \neq 0$.

Such that $7 - \sqrt{3} = \frac{a}{b}$ [½]

Therefore, $7 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get:

$$\sqrt{3} = 7 - \frac{a}{b} = \frac{7b - a}{b}$$ [½]

Since a and b are integers, we get $7 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational. [½]

But this contradicts the fact because $\sqrt{3}$ is irrational.

So, we conclude that $7 - \sqrt{3}$ is irrational. [1]

15. We know that

$a = bq + r,\ 0 \le r < b$

Let 'a' be any integer and let 'b' = 3.

$a = 3q + r \qquad 0 \le r < 3$ [½]

The possible values of r are 0, 1, 2.

$\therefore\ a = 3q,\ 3q + 1,\ 3q + 2$ [1]

$(a)^3 = (3q)^3 = 27q^3$

$= 9(3q^3)$

$= 9k$, where $k = 3q^3$ [½]

$(a)^3 = (3q + 1)^3$

$= 27q^3 + 1 + 27q^2 + 9q$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9k + 1$, where $k = 3q^3 + 3q^2 + q$ [½]

$(a)^3 = (3q + 2)^3$

$= 27q^3 + 8 + 54q^2 + 36q$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$= 9k + 8$, where $k = 3q^3 + 6q^2 + 4q$ [½]

$\therefore$ Cube of any positive integer is of the form 9k, 9k + 1 or 9k + 8.

SECTION – D

16. Let the number of students in a column be x

Then the number of columns of girls = $\frac{1533}{x}$ [1]

and number of columns of boys = $\frac{876}{x}$ [1]

Here the number of columns should be least, hence x should have a value as high as possible and x is the common factor of 1533 and 876. [1]

x = GCD (1533, 876)

= 219 [1½]

Total number of columns = $\frac{1533}{219} + \frac{876}{219}$

7 + 4 = 11 columns [1½]